RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [BATCH 2015-18]
B.A./B.Sc. FOURTH SEMESTER (January – June) 2017
Mid-Semester Examination, March 2017

Date : 15/03/2017 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : IV Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

Answer any five questions from Question nos. 1 – 7 : [5×3]

1. Prove or disprove : If A, B are two disjoint closed sets in a metric space (X, d) then $d(A,B) > 0$.

2. Prove that every subspace of a separable metric space is separable.

3. Prove or disprove : Let A be an infinite and bounded subset of a metric space (X, d). Then A has at least one limit point.

4. Define a metric on $\mathbb{N}$, the set of all natural numbers such that no point of $\mathbb{N}$ is isolated. Justify your answer.

5. Define equivalent metrics. Give example of two metric spaces (X,d_1) and (X,d_2) such that (X,d_1) is bounded, (X,d_2) is not bounded but d_1, d_2 are equivalent.

6. Let G be a dense open set in $\mathbb{R}$ and $x \in \mathbb{R}$. Prove that there exist $a, b \in G$ such that $x = a - b$.

7. Let A be an uncountable set in $\mathbb{R}$. Show that A has a limit point.

Answer any one question from Question nos. 8 & 9 : [1×4]

8. State and prove cauchy's criterion for uniform convergence for a sequence of function. [1+3]

9. State and prove Dini's theorem for uniform convergence for a sequence of function. [1+3]

Answer any two questions from Question nos. 10 - 12 : [2×3]

10. For each $n \in \mathbb{N}$, $f_n(x) = nx,\ 0 \le x \le \frac{1}{n}$

$$= 1, \quad \frac{1}{n} < x \le 1$$

a) Show that the sequence $\{f_n\}$ converges to a function f on [0,1]
b) Show that the convergence of the sequence is not uniform on [0,1]

11. For each $n \in \mathbb{N}$, $f_n(x) = nx^2,\ 0 \le x \le \frac{1}{n}$

$$= x, \quad \frac{1}{n} < x \le 1$$

Show that the sequence $\{f_n\}$ converges uniformly on [0,1]

12. Examine uniform convergence of $\{f_n\}$ on [0,1], where $f_n(x) = \dfrac{nx}{1+n^3x^2},\ x \in [0,1]$.

Group – B

Answer __any three__ questions from __Question nos. 13 – 17__ : [3×5]

13. Solve, by the method of variation of parameters, the equation $\frac{d^2y}{dx^2}+4y=4\tan 2x$.

14. Find the eigen-values and eigen-functions of $\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0\ (\lambda>0)$, under boundary conditions $y(1)=0$ and $y(e^{\pi})=0$.

15. Solve the system of differential equations : $\frac{dx}{dt}=3x-4y$, $\frac{dy}{dt}=x-y$.

16. Solve the differential equation : $\frac{dx}{x^2+a^2}=\frac{dy}{xy-az}=\frac{dz}{xz+ay}$.

17. Examine the condition of integrability for the differential equation $(x^2+y^2+z^2)dx-2xydy-2xzdz=0$ and then solve it.

Answer __any two__ questions from __Question nos. 18 – 20__ : [2×5]

18. If $x\cos\alpha+y\sin\alpha=p$ touches the curve $\frac{x^m}{a^m}+\frac{y^m}{b^m}=1$ show that $(a\cos\alpha)^{\frac{m}{m-1}}+(b\sin\alpha)^{\frac{m}{m-1}}=p^{\frac{m}{m-1}}$.

19. Find the pedal equation of the parabola $y^2 = 4ax$ with regard to its vertex.

20. Determine the asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$.

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